CHOICE OF RHEOLOGICAL MODELS

A. Kh. Mirzadzhanzade, Z. P. Shul'man, and M. G. Kopeikis

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One often finds that it is difficult to give preference to any of several proposed mechanical models for the behavior of anomalous liquids. It is clear that there must be some range of experimental conditions under which one can distinguish between the models. For this reason we consider the choice of experiment plan and criterion for reliable model choice to describe a given liquid.

Let n experiments be performed in constructing a rheological curve; the experimenter may describe the results on a heuristic basis via three models, which cannot be distinguished on the basis of the commonly accepted criterion of an identity measure. In that case, one must perform experiment (n + 1) under conditions that will provide an answer.

Kuhlbak proposed a discriminant function for distinguishing models, and Box and Hill [1] then produced an improved style. In the latter case use is made of the information obtained in the first n experiments. The results are used to estimate the parameters of the proposed v models and the dispersion σ^2 due to the experiments and the dispersions σ_i^2 of the proposed models. Then one calculates the ratio of the number of degrees of freedom in a model to the sum of the squares of the residuals ν_i/Φ_i in each case, which is proportional to the a priori probability $P_i^{(n)}$ as calculated from experiments for each model. Finally, one selects a vector for the conditions in experiment (n + 1) from the condition for a maximum in the discriminant function

$$K_v = \frac{1}{2}\sum_{r=1}^{v}\sum_{s=r+1}^{v} P_r^{(n)} P_s^{(n)} \left[\frac{(\sigma_r^2-\sigma_s^2)^2}{(\sigma^2+\sigma_r^2)(\sigma^2+\sigma_s^2)} + (\hat{Y}_r^{(n+1)} - \hat{Y}_s^{(n+1)})^2\left(\frac{1}{\sigma^2+\sigma_r^2}+\frac{1}{\sigma^2+\sigma_s^2}\right)\right], \tag{1}$$

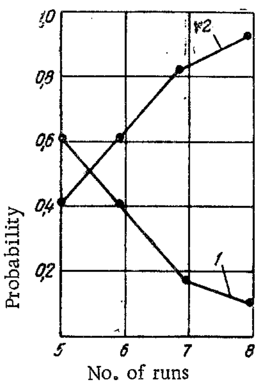

Fig. 1. Probability variation during the tests for models 1 and 2.

where $\hat{Y}_i^{(n+1)}$ is the predicted response of model i in experiment (n + 1), in which the entire procedure is repeated under these conditions. The experiment ceases when P_i becomes different, which enables one to give preference to some particular model.

Table 1 gives the typical relation between the tangential shear stress $\tau = \Delta PR/2l$ (ΔP is the pressure loss from friction, R is capillary radius, and l is capillary length) as a function of the velocity gradient $\dot{\gamma} = 4Q/\pi R^3$ (Q is the volume flow rate of the liquid) as recorded for a certain anomalous petroleum, where fresh experiments under identical conditions gave information about the residual dispersion. Two models are proposed to explain the reduced flow curve: the Shvedov–Bingham

$$\tau = \tau_0 + \eta_0\dot{\gamma} \tag{2}$$

and the modified Caisson model

$$\tau^{\frac{1}{m}} = \tau_0^{\frac{1}{m}} + (\eta_0\dot{\gamma})^{\frac{1}{m}}. \tag{3}$$

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TABLE 1

Expt.No.	$\dot{\gamma}$, 1/sec	τ, g/cm²
1	17,55	1,35
2	31,38	2,16
3	47,24	3,03
4	60,74	3,66
5	61,13	3,66
6	79,30	4,5
7	6,7	0,63
8	99,88	5,43

The parameters of models (2) and (3) were estimated from the first five experiments to give the following results respectively: $\tau_0 = 0.262$, $\eta_0 = 0.058$, and $\tau_0 = 0.683$, $\eta_0 = 0.052$, and $m = 1.92$, with equal identity measures. When K_V have been maximized, three further experiments were performed in sequence, whose results are given at the end of the Table. The revised parameters of (2) and (3) from 8 experiments were $\tau_0 = 0.289$, $\eta_0 = 0.052$ and $\tau_0 = 0.6$, $\eta_0 = 0.048$, $m = 1.8$. Then, although the identity measure for both models was 0.999, the a posteriori probabilities were $P_1^{(8)} = 0.11$ for the first model and $P_2^{(8)} = 0.89$ for the second, so after 8 experiments one can give preference to the Caisson model.

Figure 1 shows how the probabilities vary during the experiments; in this serial procedure, the number of experiments is not preset. The information provided by each experiment is entirely incorporated in the decision process. The experiments are continued until the models can be discriminated. Usually, only a few experiments are required if these are conducted in a certain critical region of the variables. If it is still impossible to discriminate between the models on increasing the number of experiments, then that number must be controlled by economic considerations.

The necessary number of experiments to discriminate models should form the subject of a separate study.

LITERATURE CITED

1. D. Himmelblau, Process Analysis via Statistical Data [Russian translation], Mir, Moscow (1973).